OCR
RECOGNISING ACHIEVEMENT

**ADVANCED GCE**

# MATHEMATICS

Core Mathematics 4

**4724**

Candidates answer on the Answer Booklet

**OCR Supplied Materials:**

- 8 page Answer Booklet
- List of Formulae (MF1)

**Other Materials Required:**

- Scientific or graphical calculator

**Friday 11 June 2010**
**Morning**

**Duration:** 1 hour 30 minutes

## INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

## INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

**1** Expand $(1 + 3x)^{-\frac{5}{3}}$ in ascending powers of $x$, up to and including the term in $x^3$. **[5]**

**2** Given that $y = \dfrac{\cos x}{1 - \sin x}$, find $\dfrac{dy}{dx}$, simplifying your answer. **[4]**

**3** Express $\dfrac{x^2}{(x-1)^2(x-2)}$ in partial fractions. **[5]**

**4** Use the substitution $u = \sqrt{x+2}$ to find the exact value of

$$\int_{-1}^{7} \frac{x^2}{\sqrt{x+2}}\, dx.$$ **[7]**

**5** Find the coordinates of the two stationary points on the curve with equation

$$x^2 + 4xy + 2y^2 + 18 = 0.$$ **[7]**

**6** Lines $l_1$ and $l_2$ have vector equations

$$\mathbf{r} = \mathbf{j} + \mathbf{k} + t(2\mathbf{i} + a\mathbf{j} + \mathbf{k}) \qquad \text{and} \qquad \mathbf{r} = 3\mathbf{i} - \mathbf{k} + s(2\mathbf{i} + 2\mathbf{j} - 6\mathbf{k})$$

respectively, where $t$ and $s$ are parameters and $a$ is a constant.

**(i)** Given that $l_1$ and $l_2$ are perpendicular, find the value of $a$. **[3]**

**(ii)** Given instead that $l_1$ and $l_2$ intersect, find

**(a)** the value of $a$, **[4]**

**(b)** the angle between the lines. **[3]**

**7** The parametric equations of a curve are $x = \dfrac{t+2}{t+1}$, $y = \dfrac{2}{t+3}$.

**(i)** Show that $\dfrac{dy}{dx} > 0$. **[6]**

**(ii)** Find the cartesian equation of the curve, giving your answer in a form not involving fractions. **[5]**

**8** **(i)** Find the quotient and the remainder when $x^2 - 5x + 6$ is divided by $x - 1$. **[3]**

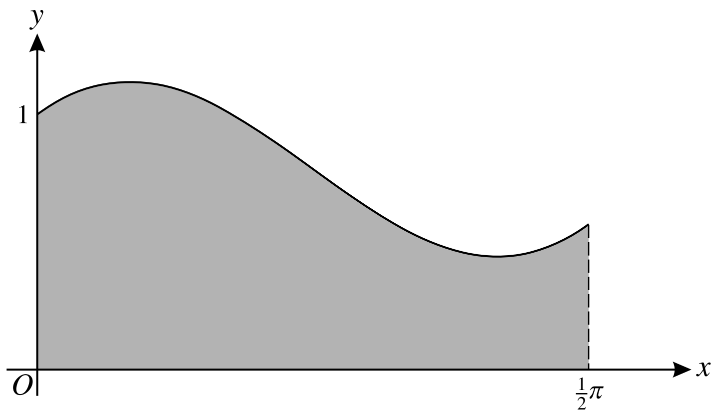
**(ii)** **(a)** Find the general solution of the differential equation

$$\left(\frac{x-1}{x^2 - 5x + 6}\right)\frac{dy}{dx} = y - 5.$$ **[3]**

**(b)** Given that $y = 7$ when $x = 8$, find $y$ when $x = 6$. **[4]**

**9** **(i)** Find $\int (x + \cos 2x)^2 \, dx$. **[9]**

**(ii)**




The diagram shows the part of the curve $y = x + \cos 2x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$. The shaded region bounded by the curve, the axes and the line $x = \frac{1}{2}\pi$ is rotated completely about the $x$-axis to form a solid of revolution of volume $V$. Find $V$, giving your answer in an exact form. **[4]**

| | | | |
|---|---|---|---|
| **1** | First 2 terms in expansion = $1-5x$ | B1 | (simp to this, now or later) |
| | $3^{rd}$ term shown as $\dfrac{-\frac{5}{3}.-\frac{8}{3}}{2}(3x)^2$ | M1 | $-\dfrac{8}{3}$ can be $-\dfrac{5}{3}-1$ <br> $(3x)^2$ can be $9x^2$ or $3x^2$ |
| | $=+\ 20x^2$ | A1 | |
| | $4^{th}$ term shown as $\dfrac{-\frac{5}{3}.-\frac{8}{3}.-\frac{11}{3}}{2.3}(3x)^3$ | M1 | $-\dfrac{11}{3}$ can be $-\dfrac{5}{3}-2$ <br> $(3x)^3$ can be $27x^3$ or $3x^3$ |
| | $=-\dfrac{220}{3}x^3$ ISW | A1 | Accept $-\dfrac{440}{6}x^3$ ISW |

N.B. If 0, SR B2 to be awarded for $1-\frac{5}{3}x+\frac{20}{9}x^2-\frac{220}{81}x^3$. Do not mark $(1+x)^{-5/3}$ as a MR.

$\boxed{5}$

| | | | |
|---|---|---|---|
| **2** | Attempt quotient rule <br> [ Show fraction with denom $(1-\sin x)^2$ & num $+/-(1-\sin x)+/-\sin x+/-\cos x+/-\cos x$ ] | M1 | |
| | Numerator = $(1-\sin x).-\sin x-\cos x.-\cos x$ <br> { Product symbols must be clear or implied by further work } | A1 | terms in any order |
| | Reduce correct numerator to $1-\sin x$ | B1 | or $-\sin x+\sin^2 x+\cos^2 x$ |
| | Simplify to $\dfrac{1}{1-\sin x}$ ISW | A1 | Accept $-\dfrac{1}{\sin x-1}$ |

$\boxed{4}$

| | | | |
|---|---|---|---|
| **3** | $\dfrac{A}{x-1}+\dfrac{B}{(x-1)^2}+\dfrac{C}{x-2}$ | M1 | For correct format |
| | $A(x-1)(x-2)+B(x-2)+C(x-1)^2\equiv x^2$ | M1 | |
| | $A=-3$ | A1 | |
| | $B=-1$ | A1 | (B1 if cover-up rule used) |
| | $C=4$ | A1 | (B1 if cover-up rule used) |

[NB1: Partial fractions need not be written out; correct format + correct values sufficient.

NB2: Having obtained $B$ & $C$ by cover-up rule, candidates may substitute into general expression & algebraically manipulate; the M1 & A1 are then available if deserved.]

$\boxed{5}$

| These special cases using different formats are the only other ones to be considered | | Max |
|---|---|---|
| $\dfrac{A}{x-1}+\dfrac{Bx+C}{(x-1)^2}+\dfrac{D}{x-2}$ ; | M1 M1; A0 for any values of $A$, $B$ & $C$, A1 or B1 for $D=4$ | 3 |
| $\dfrac{Ax+B}{(x-1)^2}+\dfrac{C}{x-2}$ ; | M0 M1; A1 for $A=-3$ <u>and</u> $B=2$, A1 or B1 for $C=4$ | 3 |

| | | | | |
|---|---|---|---|---|
| 4 | | Att by diff to connect d$x$ & d$u$ or find $\frac{dx}{du}$ or $\frac{du}{dx}$ (not d$x$=d$u$) | M1 | no accuracy; not 'by parts' |
| | | d$x$ = 2$u$ d$u$ or $\frac{du}{dx}=\frac{1}{2}(x+2)^{-\frac{1}{2}}$ AEF | A1 | |
| | | Indefinite integral $\to \int 2(u^2-2)^2\,(\frac{u}{u})\,(du)$ | A1 | May be implied later |
| | | {If relevant, cancel u/u and} attempt to square out<br>{dep $\int k\mathrm{I}\,(du)$ where $k=2$ or $\frac{1}{2}$ or 1 and I = $(u^2-2)^2$ or $(2-u^2)^2$ or $(u^2+2)^2$ } | M1 | |
| | | Att to change limits if working with f($u$) after integration | M1 | or re-subst into integral attempt and use −1 & 7 |
| | | Indef integ = $\frac{2}{5}u^5 +/- \frac{8}{3}u^3+8u$ or $\frac{1}{10}u^5 +/- \frac{2}{3}u^3+2u$ | A1 | or $\frac{1}{5}u^5 +/- \frac{4}{3}u^3+4u$ |
| | | $\frac{652}{15}$ or $43\frac{7}{15}$ ISW but no '+c' | A1 | |
| | | | $\boxed{7}$ | |
| 5 | | $\frac{d}{dx}(xy)=x\frac{dy}{dx}+y$ s.o.i. | B1 | Implied by e.g., $4x\frac{dy}{dx}+y$ |
| | | $\frac{d}{dx}(y^2)=2y\frac{dy}{dx}$ | B1 | |
| | | Diff eqn(=0 can be implied)(solve for $\frac{dy}{dx}$ and ) put $\frac{dy}{dx}=0$ | M1 | |
| | | Produce only $2x+4y=0$ (though AEF acceptable ) | *A1 | without any error seen |
| | | Eliminate $x$ or $y$ from curve eqn & eqn(s) just produced | M1 | |
| | | Produce either $x^2=36$ or $y^2=9$ | dep* A1 | Disregard other solutions |
| | | $(\pm 6, \mp 3)$ AEF, as the only answer ISW | dep* A1 | Sign aspect must be clear |
| | | | $\boxed{7}$ | |
| 6 | **(i)** | State/imply scalar product of any two vectors = 0 | M1 | |
| | | Scalar product of correct two vectors = 4 + 2$a$ −6 | A1 | ($4+2a-6=0 \to$ M1A1) |
| | | $a=1$ | A1 **3** | |
| | **(ii)** | **(a)** Attempt to produce at least two relevant equations | M1 | e.g. $2t=3+2s$ ….. |
| | | Solve two not containing '$a$' for $s$ and $t$ | M1 | |
| | | Obtain at least one of $s=-\frac{1}{2}$, $t=1$ | A1 | |
| | | Substitute in third equation & produce $a=-2$ | A1 **4** | |
| | | **(b)** Method for finding magnitude of any vector | M1 | possibly involving '$a$' |
| | | Using $\cos\theta=\frac{\mathbf{a.b}}{\lvert\mathbf{a}\rvert\lvert\boldsymbol{b}\rvert}$ for the pair of direction vectors | M1 | possibly involving '$a$' |
| | | 107, 108 (107.548) or 72, 73, 72.4, 72.5 (72.4516) c.a.o. | A1 **3** | 1.87, 1.88 (1.87707) or 1.26 |
| | | | $\boxed{10}$ | |

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|---|---|---|---|---|
| 7 | (i) | Differentiate $x$ as a quotient, $\dfrac{v\,\mathrm{d}u - u\,\mathrm{d}v}{v^2}$ or $\dfrac{u\,\mathrm{d}v - v\,\mathrm{d}u}{v^2}$ | M1 | or product clearly defined |
| | | $\dfrac{\mathrm{d}x}{\mathrm{d}t} = -\dfrac{1}{(t+1)^2}$ or $\dfrac{-1}{(t+1)^2}$ or $-(t+1)^{-2}$ | A1 | WWW $\rightarrow$ 2 |
| | | $\dfrac{\mathrm{d}y}{\mathrm{d}t} = -\dfrac{2}{(t+3)^2}$ or $\dfrac{-2}{(t+3)^2}$ or $-2(t+3)^{-2}$ | B1 | |
| | | $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\frac{\mathrm{d}y}{\mathrm{d}t}}{\frac{\mathrm{d}x}{\mathrm{d}t}}$ | M1 | quoted/implied and used |
| | | $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{2(t+1)^2}{(t+3)^2}$ or $\dfrac{2(t+3)^{-2}}{(t+1)^{-2}}$ (dep 1st 4 marks) | *A1 | ignore ref $t=-1, t=-3$ |
| | | State <u>squares</u> +ve or $(t+1)^2$ & $(\mathrm{t}+3)^2$ + ve $\therefore \dfrac{\mathrm{d}y}{\mathrm{d}x}$ +ve | dep*A1 **6** | or $\left(\frac{t+1}{t+3}\right)^2$ + ve . Ignore $\geq 0$ |
| | (ii) | Attempt to obtain $t$ from either the $x$ or $y$ equation | M1 | No accuracy required |
| | | $t = \dfrac{2-x}{x-1}$ AEF <u>or</u> $t = \dfrac{2}{y} - 3$ AEF | A1 | |
| | | Substitute in the equation not yet used in this part | M1 | or equate the 2 values of $t$ |
| | | Use correct meth to eliminate ('double-decker' ) fractions | M1 | |
| | | Obtain $\underline{2x + y = 2xy + 2}$ ISW AEF | A1 **5** | but not involving fractions $\boxed{11}$ |
| 8 | (i) | <u>Long division method</u> | | <u>Identity method</u> |
| | | Evidence of division process as far as 1st stage incl sub | M1 | $\equiv Q(x-1) + R$ |
| | | (Quotient = ) $x-4$ | A1 | $Q = x-4$ |
| | | (Remainder =) 2 ISW | A1 **3** | $R = 2$ ; N.B. might be B1 |
| | (ii) | **(a)** Separate variables; $\displaystyle\int \frac{1}{y-5}\,\mathrm{d}y = \int \frac{x^2-5x+6}{x-1}\,\mathrm{d}x$ | M1 | '$\int$' may be implied later |
| | | Change $\dfrac{x^2-5x+6}{x-1}$ into their (Quotient + $\dfrac{\text{Rem}}{x-1}$) | M1 | |
| | | $\ln(y-5) = \sqrt{}$ (integration of their previous result) (+$c$)ISW | $\sqrt{}$A1 **3** | f.t. if using Quot + $\dfrac{\text{Rem}}{x-1}$ |
| | (ii) | **(b)** Substitute $y = 7$, $x = 8$ into their eqn containing '$c$' | M1 | & attempt '$c$' ( $-3.2$ , $\ln\frac{2}{49}$ ) |
| | | Substitute $x = 6$ and their value of '$c$' | M1 | & attempt to find $y$ |
| | | <u>$y = 5.00$ (5.002529)</u> Also $5 + \frac{50}{49}e^{-6}$ | A2 **4** | Accept 5, 5.0, |
| | | Beware: <u>any</u> wrong working anywhere $\rightarrow$ A0 even if answer is one of the acceptable ones. | | |
| | | | $\boxed{10}$ | |

| | | | |
|---|---|---|---|
| **9 (i)** | Attempt to multiply out $(x+\cos 2x)^2$ | M1 | Min of 2 correct terms |
| | <u>Finding $\int 2x\cos 2x\,dx$</u> | | |
| | Use $u=2x, dv=\cos 2x$ | M1 | $1^{st}$ stage $f(x) +/- \int g(x)\,dx$ |
| | $1^{st}$ stage $x\sin 2x - \int \sin 2x\,dx$ | A1 | |
| | $\therefore \int 2x\cos 2x\,dx = x\sin 2x + \frac{1}{2}\cos 2x$ | A1 | |
| | <u>Finding $\int \cos^2 2x\,dx$</u> | | |
| | Change to $k\int +/-1+/-\cos 4x\,dx$ | M1 | where $k=\frac{1}{2}$, 2 or 1 |
| | Correct version $\frac{1}{2}\int 1+\cos 4x\,dx$ | A1 | |
| | $\int \cos 4x\,dx = \frac{1}{4}\sin 4x$ | B1 | seen anywhere in this part |
| | Result = $\frac{1}{2}x+\frac{1}{8}\sin 4x$ | A1 | |
| | **(i)** ans $=\frac{1}{3}x^3 + x\sin 2x + \frac{1}{2}\cos 2x + \frac{1}{2}x + \frac{1}{8}\sin 4x$ (+ c) | A1 **9** | Fully correct |
| **(ii)** | V = $\pi\int_0^{\frac{1}{2}\pi}(x+\cos 2x)^2\ (dx)$ | M1 | |
| | Use limits 0 & $\frac{1}{2}\pi$ correctly on their **(i)** answer | M1 | |
| | (**i**) correct value = $\frac{1}{24}\pi^3 - \frac{1}{2} + \frac{1}{4}\pi - \frac{1}{2}$ | A1 | |
| | Final answer = $\pi\left(\frac{1}{24}\pi^3 + \frac{1}{4}\pi - 1\right)$ | A1 **4** | **c.a.o.** No follow-through |
| | | $\boxed{13}$ | |

<u>Alternative methods</u>

**2** If $y=\frac{\cos x}{1-\sin x}$ is changed into $y(1-\sin x)=\cos$ x , award

M1 for clear use of the product rule (though possibly trig differentiation inaccurate)

A1 for $-y\cos x + (1-\sin x)\frac{dy}{dx} = -\sin x$ AEF

B1 for reducing to a fraction with $1-\sin x$ or $-\sin x+\sin^2 x+\cos^2 x$ in the numerator

A1 for correct final answer of $\frac{1}{1-\sin x}$ or $(1-\sin x)^{-1}$

If $y=\frac{\cos x}{1-\sin x}$ is changed into $y=\cos x(1-\sin x)^{-1}$, award

M1 for clear use of the product rule (though possibly trig differentiation inaccurate)

A1 for $\left(\frac{dy}{dx}\right)= \cos^2 x(1-\sin x)^{-2} + (1-\sin x)^{-1}.-\sin x$ AEF

B1 for reducing to a fraction with $1-\sin x$ or $-\sin x+\sin^2 x+\cos^2 x$ in the numerator

A1 for correct final answer of $\dfrac{1}{1-\sin x}$ or $(1-\sin x)^{-1}$

**6(ii)(a)** If candidates use some long drawn-out method to find '*a*' instead of the direct route, allow

M1 as before, for producing the 3 equations

M1 for any satisfactory method which will/does produce 'a', however involved

A2 for $a=-2$

**7(ii)** Marks for obtaining this Cartesian equation are not available in part (i).

If part (ii) is done first and then part (i) is attempted using the Cartesian equation, award marks as follow:

Method 1 where candidates differentiate implicitly
M1 for attempt at implicit differentiation
A1 for $\frac{dy}{dx}=\frac{2y-2}{1-2x}$ AEF
M1 for substituting parametric values of $x$ and $y$
A2 for simplifying to $\frac{2(t+1)^2}{(t+3)^2}$
A1 for finish as in original method

Method 2 where candidates manipulate the Cartesian equation to find $x=$ or $y=$
M1 for attempt to re-arrange so that either $y=f(x)$ or $x=g(y)$
A1 for correct $y=\frac{2-2x}{1-2x}$ AEF or $x=\frac{2-y}{2-2y}$ AEF
M1 for differentiating as a quotient
A2 for obtaining $\frac{dy}{dx}=\frac{2}{(1-2x)^2}$ or $\frac{(2-2y)^2}{2}$
A1 for finish as in original method

**8(ii)(b)** If definite integrals are used , then

M2 for $[\ ]_y^7=[\ ]_6^8$ or equivalent or M1 for $[\ ]_7^y=[\ ]_6^8$ or equivalent

A2 for 5, 5.0, 5.00 (5.002529) with caveat as in main scheme dep M2